ILLINOIS STATE WATER SURVEY

AT

UNIVERSITY OF ILLINOIS

URBANA, ILLINOIS

THIRD QUARTERLY TECHNICAL REPORT

April 1, 1963 - June 30, 1963

Signal Corps Grant No.:	DA-SIG-36-039^63-G2
Grant Period:	November 1, 1962 - October 3i, 1963
ARPA Order No.:	265-62-Amendraent No. 2
ARPA Project Code No.:	8900
Amount of Grant:	$48,000.00
Grantee	University of Illinois
Principal Investigators:	Professor C. D. Hendricks R. G. Semonin
Title:	Investigation of Water Droplet Coalescence

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CALCULATION OF COLLISION EFFICIENCIES OF TWO SPHERES

Previously, Lindblad and Semonin of this laboratory have calculated the collision efficiencies of two conducting spheres falling in a viscous medium with an electric field using a dipole model for the electrical force. Their hydrodynamic solution has been combined with a more sophisticated solution for the electrical forces developed by Davis [2] to provide revised collision efficiencies.

Lindblad and Semonin considered the larger of a pair of spheres to be stationary in a moving stream. Any of several methods may be used to calculate the flow about this sphere. The one used was that developed by Proudman and Pearson.[3] Their technique furnishes a stream function:

$$\psi = \frac{1}{4}(r-1)^2 \ (1-\cos^2\theta) \left[(1 + \frac{3Re}{16}) \ (2+ \frac{1}{r}) - \frac{3Re}{16} \ (2+ \frac{1}{r} + \frac{1}{r^2}) \cos\theta \right]$$

where

$Re = \frac{2\rho A V}{\mu}$ is the Reynolds number

V is the undisturbed stream velocity

A is the radius of the drop

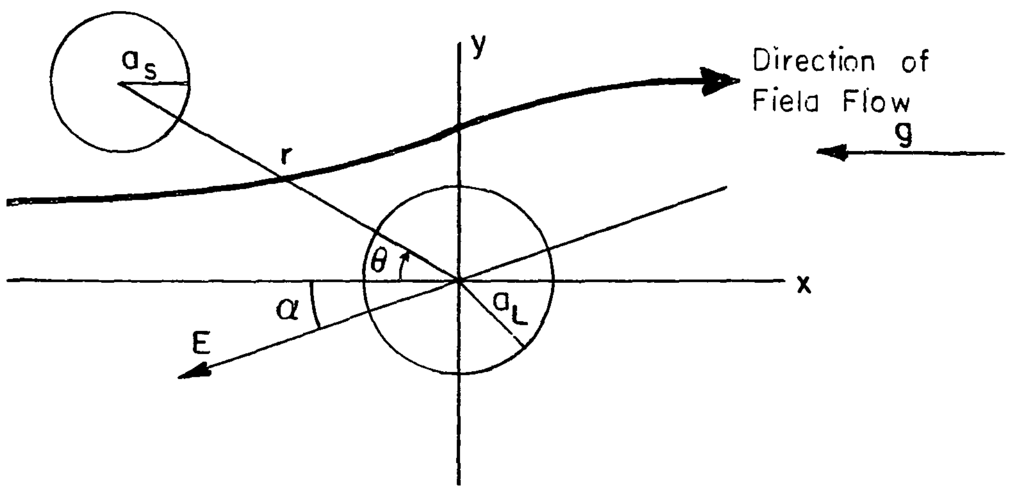
ρ is the density of the fluid

μ is the viscosity of the fluid

which can be solved for the velocity components by

$$V_r = \frac{-1}{r^2 \sin\theta} \ \frac{\partial\psi}{\partial\theta} \quad \text{and} \quad V_\theta = \frac{1}{r \sin\theta} \ \frac{\partial\psi}{\partial\theta}$$

The geometry is shown in the figure below



Where E is the electric field at angle a to the X-axis and 6 is the angle between the radius vector r and the X-axis. This solution is valid for Reynolds numbers less than two.

The equation of motion for the small sphere about the larger sphere is:

$$m_s \frac{d\vec{V}_s}{dt} = -\vec{g} + \vec{F}_D + \vec{F}_E$$

where:

m_s = mass of small sphere
$\vec{V}_s$ = velocity of small sphere
$\vec{F}_D$ = hydrodynamic forces on small sphere
$\vec{F}_E$ = electrical forces on small sphere
$\vec{g}$ = gravitational force

The linear collision efficiency definition used was:

$$CE = \frac{RS}{a_L + a_S}$$

a_L = radius of larger sphere
a_S = radius of smaller sphere

where RS is the horizontal separation of the grazing trajectory (i.e., the trajectory at which the smaller sphere just tangentially touches the larger sphere in passing). This was carried out for eight trajectories, which gave an accuracy of

$$\pm \frac{1}{2^8} = \pm 0.00390625$$

for the location of the grazing trajectory.

METHOD OF COMPUTATION: The equations of motion were integrated using the open form of the Adams-Bashforth[5] method as a predictor and the closed Adams-Bashforth method as a corrector in a finite difference scheme.

$$-y_{n+1} = y_n + n(y'_n + \tfrac{1}{2}\nabla y'_n + \frac{5}{12}\nabla^2 y'_n + \frac{3}{8}\nabla^3 y'_n + \frac{251}{720}\nabla^4 y'_n)$$

$$-y_n+1 = y_n + h(y'_{n+1} - \tfrac{1}{2}\nabla y'_{n+1} - \frac{1}{12}\nabla^2 y'_{n+1} - \frac{1}{24}\nabla^3 y'_{n+1} - \frac{19}{720}\nabla^4 y'_{n+1})$$

The method was made self-starting by including a Runge-Kutta[5] method:

$$= y_{n+1} = y_n + \frac{1}{6}(\nabla^I y_n + 2\nabla^{II} y + \nabla^{IV} y)$$

$$-\nabla^I y \equiv t(x, y)h$$

$$-\nabla^{II} \equiv t(x + \tfrac{1}{2} h,\ y + \tfrac{1}{2}\nabla^I y)h$$

$$\nabla^{III} y \equiv t(x + \tfrac{1}{2} h,\ y + \tfrac{1}{2}\nabla^{II} y)h$$

$$\nabla^{IV} y \equiv t(x + h,\ y + \nabla^{III} y)h$$

which was used for four steps to build a difference table when starting or whenever the interval of integration was changed.

RESULTS: A change in the collision efficiency at high fields was observed and is shown in the Table 1 which compares the previous results of Lindblad and Semonin.

Computation is proceeding and further details including the effect of charge on the drops will be presented in a paper in the near future.

TABLE 1

A COMPARISON OF COLLISION EFFICIENCIES FOR A 30μ AND A 5μ; DROP PAIR FOR A SIMPLE DIPOLE MODEL AND A MULTIPOLE MODEL

Horizontal Electric Field (V/cm)	Dipole	Multipole
0	0.18	0.18
300	0.18	0.32
900	0.31	0.51
1500	0.49	0.66
2100	0.62	0.79
3000	0.80	0.96
3600	0.91	1.08

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